RECREATING THE STRUCTURE OF AN OBJECT AS A WHOLE BY USING THE PROPERTIES OF ONE PART

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We consider the problem of computational diagnostics of the unknown structure of a complex object. It is assumed that one may not demonstrate a priori a model which can be used to reflect the state as a whole, but there is the possibility of an adequate description of the behavior of a specific part.

The analysis of the state of an object and likewise the synthetic modelling of its behavior is a problem which attracts constant attention. Its solution is connected with the determination of the composition and structure of factors which express the fundamental properties of the object. Subsequent formalizing and synthesis of the discovered components of an object allow one to obtain its mathematical model which is necessary for the solution of a wide range of questions.

In connection with the intensive development of methods of approach of a proposed theory of improperly posed problems [1-3], the solution of problems of analysis and synthesis based on a unified calculational—experimental study of the properties and condition of an object is unquestionably interesting.

The essence of the unification consists of the combination of a theoretical and an experiential plane of investigation of an object within the scope of a unified algorithm of the modelled analysis of the experimental state data. Its structure is expressed in the requirement to obtain a solution which on the one hand would evidently reflect the physical characteristics of the investigated object but on the other hand would be maximally consistent with the given sample of observations. It is important to note that in this case performing the formalization rests upon classical models of the Cauchy, Dirichlet, and Newton or other known types, and at the same time additional experimental information about the real operation of the object is included into the process of solution.

An expansion of the concept of the solution significantly extends the boundaries of application of known mathematical models. Using the proposed method of approach along with the theory of inverse problems, it is possible to determine not only the state of the object, but also its characteristics which are inaccessible to direct observation. As a result the object may be studied on the basis of traditional model concepts for which the formalisms will be subjected not to observations not built into the scope of the model at hand but to the model itself which inadequately describes the experiment. A similar expansion of the concept of a solution and the possibilities which appear in that case allow one to approach known problems of modelling in a new way.

We examine the problem of analyzing the unknown structure of an object from the viewpoint of the possibility of a calculational—experimental approach.

The initial task of the analysis, as is known, is the decomposition of the object into a few component elements. The performed decomposition assumes the successive independent investigation of isolated particles under the conditions of substitution of all of the remaining elements by the appropriate equivalent linkages.

The fundamental feature of the decomposition of the object is the guarantee of an adequate concept of the linkages introduced into the examination. Here one must keep in mind that in many cases there is no possibility of conducting direct experimental observations under conditions of interactions between elements of the object. In addition there may be difficulties with the direct theoretical description of the linkages of interest. In virtue of the dis-

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cussed characteristics of the decomposition of an object having a complex structure of interrelated elements, one encounters the determined methodological difficulties. In order to overcome them we propose the following approach.

Let us assume that during conducting the investigation of the structure of a complex object it is possible to obtain experimental information about the behavior of a given part of it. For this we will assume there is a known adequate model of the state of this part. Let it likewise be assumed that in the model there are several additional and, generally speaking, unknown terms, which reflect the performed decomposition and formalization of the object. Then we examine the problem of determining the structure of an object resulting from an analysis of the properties of the identified linkages of the observed part with the remaining elements of the object.

The problem of restoring the whole by analyzing the properties of the part of an object interests us and requires the substantiation of the possibility of reconstructing the general structure of the object according to limited given information about it.

The uniqueness of the determination of the source as a function of the observed effect rests upon the conditions of mutually single valued correspondence between the given and the sought elements. When such conditions of the properties of an object are satisfied, on the one hand they single valuedly determine the linkages between its parts, but on the other hand there is the uniqueness of the inverse transformation, when according to the form of the given linkages one may unambiguously judge the properties of the object.

A range of general questions in this scheme is examined in [4-6]. In particular, for linear operator models it is shown in [5] that for finding all of the parameters of the mathematical model it is sufficient to carry out a single experiment. More generally, non-linear models may be studied in terms of their determinability on the basis of the method which is presented in [6].

A definite answer to the question about achieving the maximum information content when processing the observations even for a limited amount of information makes it possible to reconstruct for specific conditions all linkages of a separated element. Then, based upon the conditions of the determinability of the desired linkages, one may investigate the specified picture of the structure of the environment of the isolated element. The information obtained expresses the interconnection conditions between elements of the object and consequently reflects its characteristic properties. This allows one to demonstrate the directions of further modelling and to carry out the identification of new linkages. A subsequent transition from the investigation of individual elements of an object to the exhibition of its given composition allows one to establish the form of the sought structure of the object.

We remark that in the case when there is significant uncertainty of choice of the form of the sought model it may be advisable to conduct an alternative modelling [7]. With its help and from the condition of consistency with all the performed interpretations of the given observations, a significant filtration of modelling errors is possible. In addition, the additional information about the physical characteristics of the investigated object likewise allows one to bound the identification by locating a single element [8].

We will show the application of the described approach by an example investigating significant factors of a specific complex heat exchange, for which the accessible observations prove to be only those across a limited part of its state.

We will study the behavioral characteristics of superfluid helium (He-II) in a narrow duct to which a powerful local heat pulse is applied.

The complications of a detailed experimental study of such thermal objects are the significant difficulties of recording the phase changes of a cryogen in the cross section of a duct. On the other hand, mathematical modelling of the boiling process of He-II in a duct does not have a complete theoretical description [9] at this time. The available information in this case are the data about the temperature field along the axis of the duct. This is connected with the problem of understanding the leakage processes during cooling of the duct from the results of observations at the temperature of the cryogen in it. The solution of such a problem presents not only methodological interest, but also has practical significance. Such a study of the dynamics of superheating He-II allows one to establish necessary recommendations for a range of projected energy systems [10].

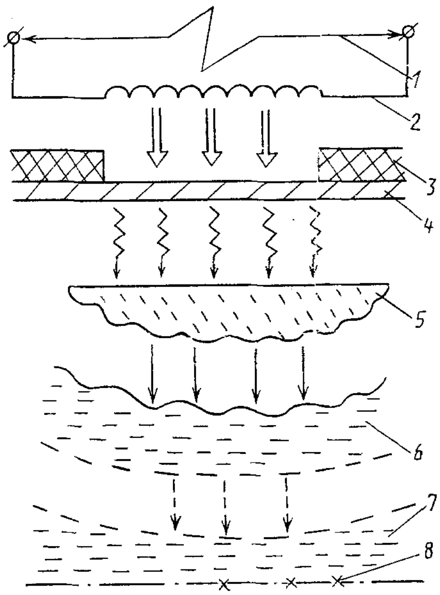

Fig. 1. Decomposition of the investigated object. 1 is the voltage source, 2 is the heater, 3 is the insulation, 4 is the duct, 5 is the vapor layer, 6 is normal helium, 7 is suerflid helium and 8 are the transducers.

We will determine the characteristics of the pulsed energy transfer by a limited volume of cryogen based on the possibility of decomposing the investigated thermal object into parts (see Fig. 1). In this case we will assume that one of its components (superheated helium) can have its temperature measured and its state may described with an adequate mathematical model. Then, based upon an identification of the thermal coupling of the isolated element with the remaining part of the object, we will establish the influence directly acting on He-II. By analyzing the properties of the discovered linkage we will determine the character of the processes producing the observed temperature field.

We carry out a processing of experimental data [11] which consists in the results of a temperature observation of the cryogen along the axis of a long thermally isolated duct without convection. The sample $\{T_{ij}^{\delta}\}_{i=\overline{1,m}}^{j=\overline{1,n}}$ will be obtained with m = 7 points along the axis of the duct for n = 5 moments in time with a measurement error no larger than δ = 0.002 K.

Separating the object into individual parts (Fig. 1) and assuming that the mechanism of the mutual friction of Gorter–Mellink [12] adequately describes the observed state of He-II, we obtain the following mathematical model:

$$c\rho\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left[\lambda\left(\frac{\partial T}{\partial x}\right)^{\frac{1}{3}}\right]+\frac{4}{\pi d^2 l}Q,\ 0<x<\frac{L}{2},\ t>0;$$

$$T|_{t=0}=T_{\text{B}},\ 0<x<L/2;\ \frac{\partial T}{\partial x}\bigg|_{x=0}=0,\ T|_{x=L/2}=T_{\text{Б}},\ t>0, \tag{1}$$

where Q is a function of the form

$$Q=\begin{cases}Q(t), & \text{if}\quad 0<x<l;\\ 0, & \text{if}\quad x\geqslant l,\end{cases}$$

which expresses the heat flux input directly into the He-II.

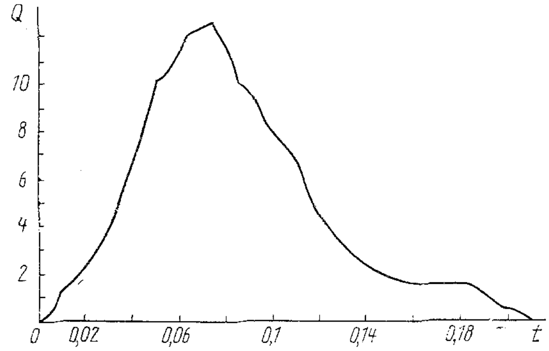

Fig. 2. The heat flux entering the interphase boundary of superheated helium. The axes Q and t are watts and seconds, respectively.

We note that the model (1) includes a term connected only with the thermal conductivity in superheated helium. The remainder of the components of the object, the occurrence of which is determined by processes occurring for absorption of the pulse by the cryogen (Fig. 1), are substituted by a given thermal coupling in the form of the function Q(t). The latter is unknown and will be subject to subsequent analysis. For the model (1) we consider that there is only a local deterioration of the one dimensionality of the temperature field in the object, which on the whole is insignificant for its determination as a multidimensional state. The allowable level of methodological error of the modelling and identification will be established in what follows.

Thus we are interested in the problem of determining the character of the processes occurring in the cryogen as a whole, and it is reduced to the solution of an inverse problem posed only for the superfluid component. Within the scope of this formulation the desired properties of the interaction of He-II with a solid body are expressed directly by the function Q(t), and analysis of the function behavior may be indicative of the natures of the leakage processes.

The solution of the inverse problem at hand, that is, finding the nature of the functions {T(x, t), Q(t)}, will be performed by the method of regularization according to the scheme of partial correspondence [13]:

$$\inf_{Q\in A}\Omega\{Q\};\ \max_{1<j<n}|T^{\delta}_{ij}-T(x_i,\ t_j)|\leqslant\delta_i,\ i=\overline{1,\ m}. \tag{2}$$

Here $T(x_i, t_j)$ is the model state found from (1) for a given Q from A at corresponding observation points.

Reducing the problem (2) to the problem of an absolute minimization of the relative parameters approximating the desired flux Q(t), we determine its nodal values $\{q_k\}_{k=\overline{1,N}}$ in the class of linear polygonal functions. In accordance with [13-15], we choose the stabilizing functional from the condition of securing a maximal regularization of the region of admissible solutions $A = C^1(t)$. The error weighting function we specify so as to guarantee the minimal level of discrepancy subject to the conditions of agreement with observations. The temperature field T(x, t) will be sought in an approximation due to Bessel, obtained from (1) by a finite-difference method.

The obtained solution of the present inverse problem for the case N = 100 is shown in Fig. 2 and is presented in Table 1. The determination of a function of such a complicated form requires one to perform a special analysis of errors of identification and modelling.

The fundamental source of methodological error of identification is the expansion of the region of permissible solution of the inverse problem [1]. The necessity of finding a function with a previously unknown form requires the achievement of an as general as possible approximation of the sought quantities. This in turn leads to the situation where beginning from a certain time, for example for a specific number of nodes of the approximation, a loss of stability may encroach on the calculations due to broadening of the region of allowable solutions. In order to guarantee a continuous dependence of the solution upon the variation in the initial data, in accordance with (1) it is necessary to introduce stabilized functionals

TABLE 1. The Temperature Field in He-II from Calculated Values and Its Comparison with Experimental Observations

Time, sec	Distance from center of duct, m						
	0,0	0,1	0,2	0,3	0,4	0,5	0,6
0,0	1,822	1,822	1,822	1,822	1,822	1,822	1,822
0,1	2,118	1,857	1,829	1,825	1,824	1,823	1,823
0,2	2,164	1,904	1,844	1,831	1,827	1,852	1,824
0,5	1,951	1,935	1,884	1,855	1,842	1,835	1,830
1,0	1,871	1,871	1,869	1,865	1,857	1,849	1,842
1,5	1,849	1,849	1,849	1,848	1,847	1,845	1,843
$\max \lvert T_{ij}^{\delta} - T(x_i, t_j)\rvert$	0,0003	0,007	0,004	0,001	0,004	0,002	0,004

of the required quantities. From these functions is required the guarantee of the greatest restriction superimposed onto the region of allowable solutions [13-15], subject to conditions of agreement with observations. In those conditions, when modelling errors are added to the measurement interferences, strong regularizing may lead to a significant smoothing of the desired function manifested as a simplification of the functional form of the sought curve. On the contrary, weak regularization leads to an excessive complication of the form of the sought solution. That behavior, as a rule, occurs when an attained level of agreement with observations is preserved [13-15].

In this way, for a given level of agreement, methodical error of the identification must be kept in mind in two cases.

If the attainable level of agreement is significantly larger than the measurement interferences, for the condition of an adequate mathematical model this implies an unsatisfactory approximation of the sought function.

In those cases when the attained agreement is satisfactory, and the functional properties of the employed curve are sufficient for the approximation of the sought solution, the error of the identification will be expressed in a significant difference of the found solution from the sought value, if the actual regularization is insufficient. The latter may lead, for example, to a perturbation of the properties of monotonicity of the solution.

Since in the present case satisfactory agreement with observations was achieved (see Table 1), then it is necessary to explain whether characteristic sections of change of the monotonicity of the curve Q(t) are due to unsatisfactory regularization or whether they express the determined properties of the object. An answer to this question is given by the synthetic modelling [1] performed. It consists in obtaining the temperature field of He-II as a function of the initial data, based on the results of an estimation of the function Q(t), with a subsequent modelling procedure to determine of the desired values. A corresponding analysis showed that even when significant deterioration of the agreement with observations is disregarded, characteristic breaks on the found curve are conserved. From that one must conclude that these sections are entirely due to distinct properties of the object.

A second much more significant methodological error of the solution of the inverse problem is the modelling errors.

The assumption we used that the temperature field varies significantly only along the axis of the duct does not consider the actual radial drop in temperature. Therefore the question arises as to what extent the function Q(t) must vary if one includes the two dimensionality of the temperature field. The definitive answer to this question one may obtain by solving the problem in a two dimensional formulation. However one may make a few conclusions if one carries out the analysis of the quantity of the error under consistent conditions.

The results of the determination show that the largest error in the description of the field of such a one dimensional distribution develops after the diffusion of the heat disturbance along the length of the duct. Initially the discrepancy is basically determined by the level of interference of the measurements. At subsequent times, when the quantity of the discrepancy increases, the characteristics in the behavior of the function alredy are not observed.

In this way, the analysis of the errors of solution show that the found solution Q(t), on the one hand, satisfactorily exhibits the fundamental functional properties of the object, but on the other it does not take care of the significant errors of identification. This conclusion allows one to establish, based upon consideration of the characteristics of the behavior of the function Q(t), what occurs in the duct after a heat input is added to it.

On the whole, the behavior of the thermal flux Q(t) reflects the dynamics of the system in which variations of the thermophysical properties occur with and without liberation of heat. The points of discontinuity of the tempo of growth of the function Q(t) show the times of these transitions. Variations in the monotonicity of the behavior of the function Q(t) express a smooth change of the regimes in which the processes of heat transfer occur. They occur both as a result of a nonabrupt change of the intrinsic properties of the object as well as due to a change in the amount of external inputs. We note that the determined function Q(t) allows one to study not only the qualitative properties of the object but also to obtain the entire range of its quantitative characteristics. For example, from the amount of heat absorbed during formation of a helium vapor film one may estimate the film's thickness.

Performing further modelling of the thermal state of helium, one may convert to the representation of an object in the form of a specific composition of its found components. For example, proceeding from the obtained results, one may consider the cryogen as the two phase system He-II-He-I. Repeating identification in that case, we determine the heat flux which is input even into the normal helium. The latter allows one to estimate the character of superheating of the wall of the duct.

The investigation we conducted shows significant potential possibilities of the methodology of inverse problems. They allow one to carry out an adequate analysis of the properties of an object under conditions when the amount of initial information is significantly limited. As is shown, when specific conditions of identifiability are fulfilled one may recover the properties of the object as a whole from data about the state of an isolated element. In particular, we show the possibility of a calculational—experimental study of the dynamics of a multiphase system by using a single phase model.

NOTATION

T(x, t) is the temperature field; x is the spatial coordinate; t is time; T^δ is the sample of study or observation; δ is the absolute error; L is the length of the duct; d is its diameter; ℓ is the length of heatable section; c is the heat capacity of He-II; ρ is the density of He-II; λ is the thermal conductivity of He-II; Q is the interphase heat flux; Ω is the stabilizing functional; A is the region of allowable solutions.

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